PHY121 Formula Sheet

Equations of Motion

One Dimension	$\bar{v} = \frac{x_f - x_i}{t_f - t_i}$ $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$ $v = v_0 + at$ $x - x_0 = v_0 t + \frac{1}{2}at^2$ $x - x_0 = \frac{v^2 - v_0^2}{2a}$ $x_f = x_i + v_i t + ½at^2$ $v_f^2 = v_i^2 + 2a(x_f - x_i)$

Projectile Motion	$v_x = v_0 \cos\theta$ $x = (v_0 \cos\theta)\, t$ $x_{max} = \frac{v_0^2 \sin 2\theta}{g}$ $v_y = v_0 \sin\theta - gt$ $y = (v_0 \sin\theta)\, t - ½ gt^2$ $y_{max} = \frac{v_0^2 \sin^2\theta}{2g}$

Rotational	$a_r = \frac{v^2}{r}$ $a = a_r + a_t$

Circular	From Newton's 2nd Law $\Sigma F_c = m\frac{v^2}{r}$

Variables	x_i = initial position	v_i = initial velocity
	x_0 = initial position	v_0 = initial velocity
	x_f = final position	v_f = final velocity
	x = displacement in the x-direction	v = velocity
	y = displacement in the y-direction	v_x = velocity in the x-direction
	t_i = initial time	v_y = velocity in the y-direction
	t_f = final time	a = acceleration
	t = time	a_r rotational acceleration
	θ = angle between the positive x-axis & velocity vector	a_t = tangential acceleration
	g = gravity (9.8 m/s² or 32 ft/s²)	r = radius
	m = mass	F_c = centripetal force

Laws of Motion	Newton's Laws An object in motion tends to stay in motion $(a = 0 \rightarrow \Delta v = 0)$ For every action there is an equal and opposite reaction $(\vec{F}_{12} = -\vec{F}_{21})$ Force equals mass times acceleration $(\Sigma\vec{F} = m\vec{a})$ Friction force = coefficient of friction times mass times the normal force Kinetic Friction: $F_{fr} = \mu_k F_N$ Static Friction: $F_{fr} \leq \mu_s F_N$

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Work and Energy Equations

Work

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

$$W_{spr} = \int_{x_i}^{x_f} F_{spr}\, dx = \int_{-x_m}^{0} (-kx)\, dx = \frac{1}{2} kx_m^2 = \tfrac{1}{2}kx_i^2 - \tfrac{1}{2}kx_f^2$$

Energy

Kinetic $K = \tfrac{1}{2}mv^2$ — Potential $U = mgh$ — Total Energy = K + U

Power $\overline{P} = \dfrac{W}{\Delta t}$ — $P = \lim_{\Delta t \to 0} \dfrac{W}{\Delta t} = \dfrac{dW}{dt}$ — $P = F \cdot \dfrac{ds}{dt} = F \cdot v$

Collisions

Momentum $p = mv$ — $F = \dfrac{dp}{dt}$ — For an isolated system, $\Delta p = 0$

Impulse $I = \int_{t_i}^{t_f} F dt = \Delta p$

Elastic (bounce) $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

Inelastic (stick) $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)v_f$

Variables

- W = work
- F = force
- F_x = force in x-direction
- s = distance
- θ = angle
- x_i, y_i, z_i = initial position in the x, y, or z direction
- x_f, y_f, z_f = final position in the x, y, or z direction
- P = power
- Δt = change in time
- p = momentum
- Δp = change in momentum
- m_1 = mass of first object
- m_2 = mass of second object
- v_{1i} = initial velocity of first object
- v_{1f} = final velocity of first object
- v_{2i} = initial velocity of second object
- v_{2f} = final velocity of second object
- W_{spr} = work done by spring
- F_{spr} = force of spring
- k = spring constant
- x = distance spring stretched/compressed
- x_m = distance spring stretched/compressed
- K = kinetic energy
- m = mass
- v = velocity
- U = potential energy
- h = height
- I = impulse
- t_i = initial time
- t_f = final time

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Rotational Motion

General Equations	s = rθ $\overline{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i}$ $\varpi = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$ $\overline{\alpha} = \frac{\omega_f - \varpi_i}{t_f - t_i}$ $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\varpi}{dt}$
Equations of Motion	$\omega = \omega_0 + \alpha t$ $\theta = \theta_0 + \omega_0 t + ½\alpha t^2$ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
Converting Rotational to Linear	$v = r\omega$ $a_t = r\alpha$ $a_r = \frac{v^2}{r} = r\omega^2$
Inertial Moment	$I = \sum m_i r_i^2$ $I = \lim_{\Delta m_i \to 0} \sum r_i^2 \Delta m_i = \int r^2 dm$ $\rho = \frac{dm}{dV}$ For Rods: $dm = \frac{M}{L}dx$ For Cylinders: $dV = dA \cdot L = (2\pi r dr)L$ Parallel Axis Theorem $I = I_{CM} + MD^2$
Torque	$\tau = r F \sin\varphi = F \cdot D$ $\tau = (mr\alpha)r = mr^2\alpha$ $\tau = I\alpha$
Other	Work: $dW = F \cdot ds = (F \sin\phi) r d\theta$ $v_{CM} = r\omega$ $a_{CM} = r\alpha$ $\tau = R \times F = R \times \frac{dp}{dt} = \frac{dL}{dt}$ $K = ½I_{CM}\omega^2 + ½Mv_{CM}^2$ = rotational+translational $l = R \times p = mvR \sin\varphi = I\omega$
Variables	s = arc length; ω = angular speed; α = angular acceleration r = radius; ω_i = initial angular speed; $\overline{\alpha}$ = average angular acceleration θ = angle; ω_0 = initial angular speed; I = Inertia θ_i = initial angle; ω_f = final angular speed; m_i = mass of i^{th} particle θ_f = final angle; $\overline{\omega}$ = average angular speed; M = mass of an object t_i = initial time; r_i = distance from i^{th} particle to axis of rotation t_f = final time; a_r = radial acceleration; D = distance from axis v = linear velocity; a_t = tangential acceleration; K = kinetic energy ρ = density of an object; L = length of rod / cylinder; φ = angle I_{cm} = Inertia about an axis through the center of mass of an object τ = torque; l = instantaneous angular momentum F = force; R = instantaneous position vector a_{CM} = acceleration at center of mass; p = instantaneous linear momentum V_{CM} = velocity at center of mass

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Miscellaneous Formulas

Center of Mass	$X_{CM} = \frac{\sum m_i x_i}{\sum m_i}$ $r_{CM} = \frac{1}{M}\int r\,dm$

Rocketry	$(M + \Delta m)v = M(v + \Delta v) + \Delta m(v - v_e)$ $M\Delta v = \Delta m v_e$ $M dv = -v_e dM$ $\int_{v_i}^{v_f} dv = -v_e \int_{M_i}^{M_f} \frac{dM}{M}$ $v_f - v_i = v_e \ln\left(\frac{M_i}{M_f}\right)$

Gravity	Law of Gravitation	$F_g = G\frac{m_1 m_2}{R^2}$ $F_g = -G\int \frac{dm}{r^2}\hat{r}$
	Kepler's Laws	$\frac{dA}{dt} = \frac{L}{2m} = \text{constant}$ $T^2 = \left(\frac{4\pi^2}{Gm}\right)r^3 = kr^3$
	Energy	$\Delta U = -Gm_1 m_2 \frac{1}{\Delta r}$ $E = \frac{Gm_1 m_2}{2r} - \frac{Gm_1 m_2}{r} = -G\frac{m_1 m_2}{2r}$

Fluids	Pressure	$p = \frac{F}{A}$ Buoyant force = weight of displaced liquid
	Bernoulli's Equation	$p + \frac{1}{2}\rho v^2 + \rho g y = c$

Variables	X_{CM} = center of mass in x-direction	v = velocity
	m_i = mass of i[th] particle	M = mass of rocket and fuel
	x_i = position of i[th] particle	Δm = mass of fuel loss
	r_{CM} = radial center of mass	v_e = exhaust speed
	M = mass	Δv = change in velocity of rocket
	m = mass	M_i = initial mass of rocket and fuel
	m_1 = mass of 1[st] object	M_f = final mass of rocket and remaining fuel
	m_2 = mass of 2[nd] object	v_i = initial velocity
	F_g = force of gravity	v_f = final velocity
	G =universal gravitational constant(6.673×10^{-11} N·m²/kg²)	R = distance seperating m_1 & m_2
	L = angular momentum of planet (constant)	$\hat{r}$ = unit vector
	T = period of revolution	r = radius
	k = constant (2.97×10^{-19} s²/m³)	Δr = change in radius
	ΔU = change in gravitational potential energy	p = pressure
	E = total energy	F = force exerted on the piston
	ρ = density (mass / volume)	A = surface area of piston
	g = gravity constant (9.8 m/s² or 32 ft/s²)	y = height
	c = constant	

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Oscillations

Simple Harmonic Motion	$x = A\cos(\omega t + \varphi)$ $\quad A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2}$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$ $\quad f = \frac{1}{T} = \frac{\omega}{2\pi}$
Energy	$K = ½ mv^2 = ½ m\omega^2 A^2 \sin(\omega t + \varphi)$ $U = ½ kx^2 = ½ kA^2\cos^2(\omega t + \varphi)$ $\quad E = K + U = ½ kA^2$
Wave Motion	$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi)$ $\quad v = \frac{dv}{dt} = -\omega^2 A\cos(\omega t + \phi)$
Waves on a string	$F_r = \frac{mv^2}{R} = 2F\sin\theta \approx 2F\theta$ $\quad m = \mu\Delta s = 2\mu R\theta$ $2F\theta = \frac{2\mu R\theta v^2}{R}$ $\quad v = \sqrt{\frac{F}{\mu}}, F = tension, \mu = \frac{kg}{m}$ $y = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$ $\quad k = angular\ \ wave\ number = \frac{2\pi}{\lambda}$ $\omega = angular\ \ frequency = \frac{2\pi}{T}$ $\quad v = \frac{dy}{dt} = -\omega A\cos(kx - \omega t)$ $a = \frac{dv}{dt} = -\omega^2 A\sin(kx - \omega t)$
Energy	$\Delta E = ½\Delta m\omega^2 A^2$ $\quad P = \frac{dE}{dt} = \frac{1}{2}\mu\omega^2 A^2 v$

Variables		
x = position	K = kinetic energy	ω = angular frequency
x_0 = initial position	U = potential energy	t = time
A = amplitude	E = total energy	φ = phase constant or phase angle
T = period	R = radius	v = velocity
m = mass	θ = angle	v_0 = initial velocity
k = spring constant	λ = wavelength	Fr = total radial force
f = frequency	t = time	F = force of tension
ΔE = change in total energy	a = acceleration	μ = mass per unit length
Δs = length of small segment of string		P = power
Δm = change in mass		